Short-time critical dynamics for the transverse Ising model

M. Santos*
Departamento de Física, Universidade Federal de Santa Catarina, 88040-900 Florianópolis, SC, Brazil
(Received 15 December 1999)

We have analyzed the short-time critical behavior for the one-dimensional quantum transverse Ising model through Monte Carlo simulations at zero temperature. We used the scaling relation for the dynamics at the early time stages in order to obtain the static critical exponents (β,ν) and the dynamical critical exponent z for this model. While the values found for the static exponents are in agreement with the exact ones, here, the dynamical critical exponent is found for a quantum spin model.

PACS number(s): 64.10.+h, 64.60.Cn, 64.60.Ht

I. INTRODUCTION

The short-time critical dynamics, recently predicted by Janssen, Shaub, and Schmittmann [1] for the $O(N)$ vector model by renormalization group arguments, has been extensively used, with success, to determine the critical exponents of various other physical systems through numerical simulations out of thermodynamic equilibrium [2–6].

The main result of this prediction is associated with a universal scaling behavior already in the initial stages of the evolution of a spin system towards the equilibrium states at the critical point. This power-law scaling form emerges when the magnetization and the correlation length of the initial configuration are zero and very small. In addition, it occurs only after a microscopic time that is small when compared with macroscopic times, but is large in the microscopic sense. However, its most surprising prediction is the critical initial increase of the magnetization, with a new independent critical exponent θ. But there is a limitation for the use of this method, the short-time critical dynamics can be used only for system models in which the critical point is exactly known.

More recently, it has been numerically argued that the short-time universal behavior also occurs in systems for which the initial configuration of the spin system is completely ordered [6]. Besides, in this case, it is possible to localize the critical point.

In this work we have applied the short-time critical dynamics for the one-dimensional transverse Ising model, at zero temperature, through numerical simulation. This model is very well studied in the literature and its critical parameters are exactly known [7,8]. We have used a new Monte Carlo method, apropriate for the investigation of the ground-state properties of quantum spin systems [9] in order to obtain the time evolution of the system towards the equilibrium states. This method is distinct from other Monte Carlo methods used to simulate quantum spin systems and has been extensively applied to several quantum spin models [10–14].

We show that, by applying this method to a completely ordered initial state, it is possible to obtain the static (β,ν) and the dynamical critical exponents (z) using the short-time scaling formalism. The values we obtain for the static exponents are in agreement with results found in the literature for the equilibrium states of the model. However, we believe that the value found for the dynamical critical exponent is the first estimate of this exponent for the transverse Ising model.

II. TRANSVERSE ISING MODEL

We start by defining the Hamiltonian model in which the z components of a quantum spin chain are coupled as in the usual Ising model. In addition, a transverse external field is applied on the system in order to include some noise and to avoid a completely ordered system at zero temperature. Then, we can write the following expression for the energy of the system:

$$\mathcal{H}=-\sum_i \sigma_i^z\sigma_{i+1}^z-\Gamma\sum_i \sigma_i^x, \tag{1}$$

where σ_i^x and σ_i^z are the Pauli operators of the spin i and the parameter Γ gives the strength of the transverse field. Depending on the values of Γ, the system can be found in two distinct phases, characterized by the value of the order parameter. In this model the order parameter is the longitudinal magnetization by spin M_z, defined as

$$M_z=\frac{1}{N}\sum_i \sigma_i^z, \tag{2}$$

where N is the number of spins in the system. If $\Gamma<1$, $M_z\neq 0$ and the system is in an ordered phase (ferromagnetic). Otherwise, if $\Gamma\geqslant 1$ the system is in a disordered phase (paramagnetic) and $M_z=0$. The transition between the ferromagnetic and paramagnetic phases is continuous at $\Gamma_c=1$ and it is in the same universality class as the two-dimensional Ising model, which is characterized by the following static critical exponents: $\nu=1,\beta=\frac{1}{8}$, and $\gamma=\frac{3}{4}$. This critical behavior of the transverse Ising model is a consequence of the fact that quantum statistical models in d dimensions can be represented by classical statistical models in $(d+1)$ dimensions [7,8].

*Electronic address: marcio@fisica.ufsc.br

III. SCALING RELATIONS

Janssen, Schaub, and Schmittmann [1] derived a scaling relation for the relaxation of the kth moment of the magnetization $M^{(k)}(t)$ at time t, for the $O(N)$ vector model, using the ϵ expansion. Near the critical point it can be written as

$$M^{(k)}(t,\tau,L,m_0)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{1/\nu}\tau,b^{-1}L,b^{x_0}m_0), \tag{3}$$

where the kth moment of the magnetization is defined by

$$M^{(k)}(t)=\frac{1}{L^d}\left\langle\left(\sum_i \sigma_i(t)\right)^k\right\rangle. \tag{4}$$

In these equations, $\tau=(T-T_c)/T_c$ is the reduced temperature, L is the linear lattice size, d is the spatial dimension of the system, m_0 is the nonzero initial magnetization, b is the spatial scaling factor, β and ν are, respectively, the static critical exponents of the magnetization and of the correlation length, z is the dynamical critical exponent and x_0 is a new independent critical exponent, that governs the initial increase of the magnetization. The initial state is obtained from a high-temperature state after a sudden quenching. Therefore, the initial magnetization and the initial correlation length are very small. This scaling law has the same form as that of the dynamic finite-size scaling theory, valid at the thermodynamical equilibrium [15] when the initial configuration is completely ordered. Then, taking $m_0=1$, the scaling relation given by Eq. (3) becomes

$$M^{(k)}(t,\tau,L)=b^{-k\beta/\nu}M^{(k)}(b^{-z}t,b^{1/\nu}\tau,b^{-1}L). \tag{5}$$

Jaster *et al.* [6] assumed that this scaling relation is also valid for short times and used the scaling relations (3) and (5) to determine the critical exponents for the three-dimensional Ising model. In both cases the results obtained are comparable with those found in numerical simulations at equilibrium. However, relation (5) also allows us to determine the critical temperature of the model and provides more accurate results than those obtained by relation (3).

In this work we assume a relation similar to Eq. (5) for the kth moment of the longitudinal magnetization of the one-dimensional (1D) transverse Ising model already in the initial stages of the evolution, when the initial configuration of the system is completely ordered. Then, if we define the kth moment of the longitudinal magnetization, at time t, as

$$M_z^{(k)}(t)=\frac{1}{L}\left\langle\left(\sum_i \sigma_i^z(t)\right)^k\right\rangle, \tag{6}$$

we can assume the following short-time scaling relation:

$$M_z^{(k)}(t,\zeta,L)=b^{-k\beta/\nu}M_z^{(k)}(b^{-z}t,b^{1/\nu}\zeta,b^{-1}L), \tag{7}$$

where $\zeta=|\Gamma_c-\Gamma|$ is the reduced strength of the transverse field. For $k=1$ the above equation gives the time evolution of the longitudinal magnetization. In this case, if we choose $b=t^{1/z}$ for the spatial scaling factor we can show, for sufficiently large values of L, that the following scaling relation is valid:

$$M_z(t,\zeta)=t^{-\beta/\nu z}M_z(1,t^{1/\nu z}\zeta). \tag{8}$$

Therefore, at the critical point, where $\zeta=0$,

$$M_z(t)\sim t^{-\theta_1}, \tag{9}$$

where $\theta_1=\beta/\nu z$. With this relation it is possible to determine the dynamical critical exponent z if β and ν are given. However, other relations can be obtained from the scaling relation (7). For instance, taking the derivative in both sides of Eq. (8) with respect to ζ we can show that the logarithmic derivative of the longitudinal magnetization evolves in time, at the critical point, as

$$\delta_\zeta M_z(t,\zeta)|_{\zeta=0}\sim t^{\theta_2}, \tag{10}$$

where $\theta_2=1/\nu z$ and

$$\delta_\zeta=\frac{\partial}{\partial\zeta}\ln. \tag{11}$$

Another important quantity, which is used to characterize the critical point in numerical simulations at equilibrium, is the Binder's cumulant. Here, we have defined a similiar quantity using the second moment of the order parameter

$$U_z(t)=\frac{M_z^{(2)}(t)}{[M_z(t)]^2}-1. \tag{12}$$

Within the short-time scaling arguments we can show that

$$U_z(t)\sim t^{\theta_3}, \tag{13}$$

where $\theta_3=d/z$ and d is the spatial dimension of the system.

Therefore, we have three dynamical scaling relations for three unknown exponents. That is, through Eqs. (9), (10), and (12) we can determine each one of the exponents z,β, and ν. Although the $1D$ transverse Ising model presents an exact solution at equilibrium, we cannot obtain the exact value of the dynamical critical exponent z. Only for the dynamical Ising model in one dimension is it possible to find the exact value of z, as was shown by Glauber [16] almost 40 years ago. To the best of our knowledge z has not been obtained for a quantum spin model. In this work we have performed Monte Carlo simulations to determine the initial relaxation towards equilibrium and to obtain the related exponents $\theta_i, i=1,2$, and 3.

IV. MONTE CARLO METHOD AND NUMERICAL RESULTS

The Monte Carlo method used in this work was specially introduced in order to study the ground-state properties of quantum spin systems [9]. The method is based on the fact that statistical quantum models in d dimensions can be mapped into the statistical classical models in $d+1$ dimensions [7,8]. In addition, it takes into account that a matrix with non-negative elements can be regarded as a transfer matrix of a classical statistical model, with the value of the leading eigenvalue and its eigenvector providing the statistical properties of the model. Then, this Monte Carlo algorithm gives information about the leading eigenvector.

The method is implemented as a Markov process, in which the stationary probability of a given configuration is

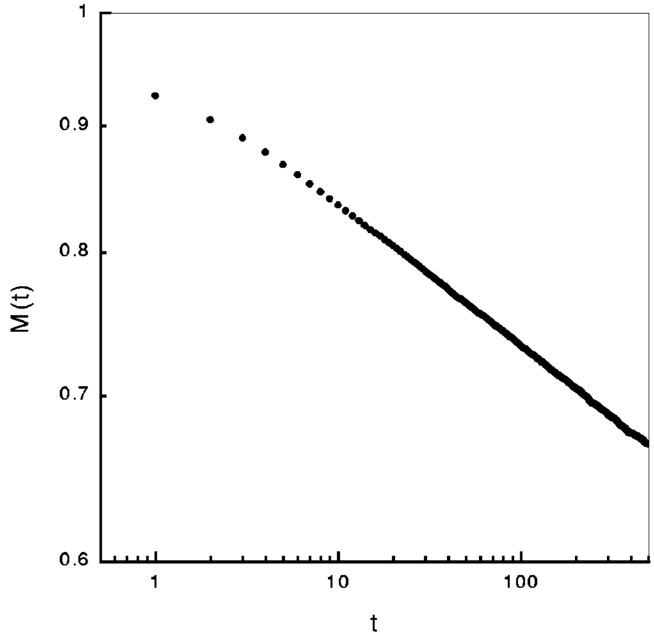

FIG. 1. The log-log plot of the magnetization vs time for the two-dimensional Ising model at the critical value of the temperature.

written as a product of transfer matrices. The transition probability between two configurations obeys the detailed balance condition and the system evolves in time according to the Metropolis prescription. A complete description of this method, applied to the one-dimensional transverse Ising model can be found in Ref. [13].

In this work we have performed simulations for chains of linear size $L=256$ over 1000 samples and we have measured the quantities given by Eq. (6), for $k=1$ and $k=2$, at the critical point $\Gamma_c=1$. In order to obtain the time evolution of the logarithmic derivative of the magnetization we have also performed simulations at $\Gamma=\Gamma_c\pm0.005$, and we have used the quadratic interpolation algorithm to do the derivative at each time t. As usual, the time unit is given in Monte Carlo step (MCS), which is defined by a complete update of the spins of the system.

At this point a comment about the microscopic time is

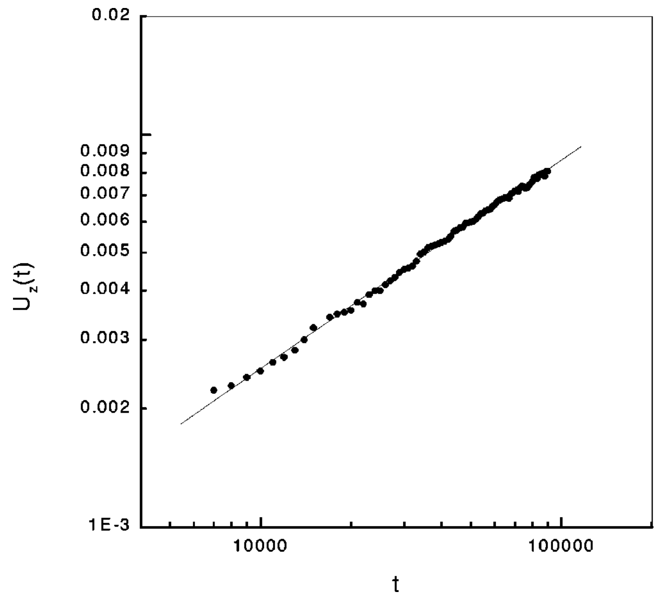

FIG. 3. Time evolution of the Binder's cumulant $U_z(t)$, on a log-log scale, for the one-dimensional transverse Ising model at the critical value of the amplitude of the transverse field. The points represent the MC data and the continuous line is the best fit to the data points.

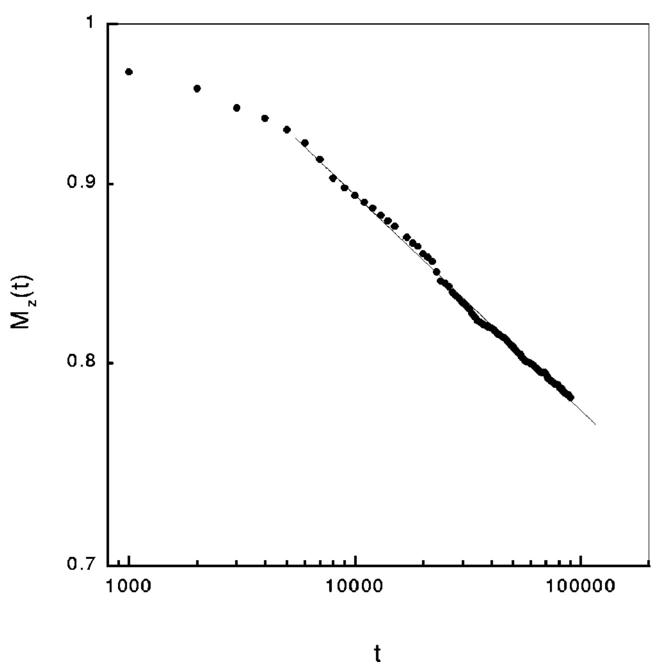

FIG. 2. Time evolution of the longitudinal magnetization $M_z(t)$, on a log-log scale, for the one-dimensional transverse Ising model at the critical value of the amplitude of the transverse field. The points represent the MC data and the continuous line is the best fit to the data points.

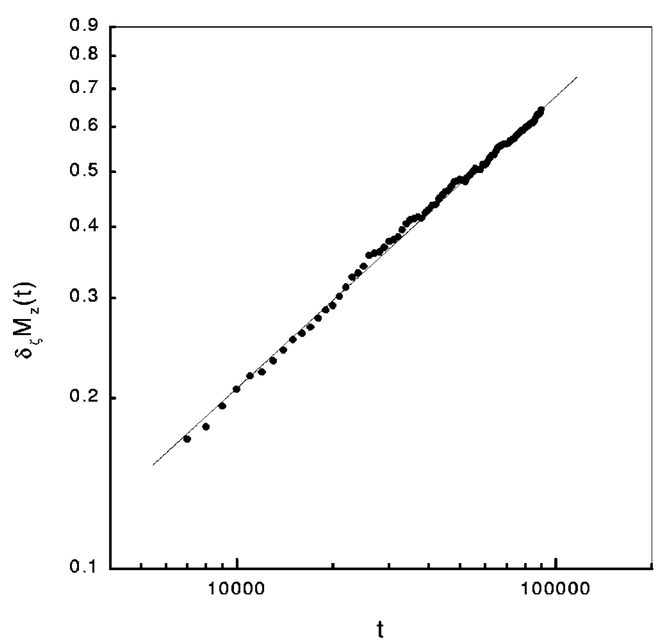

FIG. 4. The log-log plot of the logarithmic derivative of the longitudinal magnetization with respect to ζ vs time for the one-dimensional transverse Ising model at the critical value of the amplitude of the transverse field. The points represent the MC data and the continuous line is the best fit to the data points.

necessary. Different from the simulations of the two-dimensional Ising model, the microscopic time in our model is not very small. In fact, we can see in Figs. 1 and 2 the time evolution of the order parameter, represented by the points, for both models. Whereas for the Ising model the microscopic time is smaller than 10 MCS, for the transverse Ising model it is about 7000 MCS. Therefore only after this value of time the longitudinal magnetization exhibits a power-law scaling form. Then, the simulations were carried out for times up to 10^5, and the plotted quantities were analyzed for $7\times10^3<t<1\times10^5$ MCS, at time intervals of 10^3 MCS.

Now, we investigate the short-time behavior of the transverse Ising model in order to obtain the critical exponents. In Fig. 2 we have plotted, on the log-log scale, the longitudinal magnetization vs time. The points represent the MC data, while the full line gives the best fit to the data points. The slope of this line is $\theta_1=0.0616\pm0.0004$. The time evolution of the dynamic Binder's cumulant is exhibited in Fig. 3 on a log-log scale. From the best fit to the data points we obtain $\theta_2=0.531\pm0.002$. Finally, in Fig. 4 we present the log-log plot for the logarithmic derivative of the longitudinal magnetization as function of time. The slope of the full line is $\theta_3=0.514\pm0.003$. Therefore, with these values, and using Eqs. (9), (10), and (12), it is possible to show that the exponents β, ν, and z are given by

$$\beta=0.119\pm0.002,$$

$$\nu=1.03\pm0.01,$$

and

$$z=1.883\pm0.007.$$

The relative errors in the values of the static exponents are less than 5%, when compared with their exact values. The value found for the dynamical critical exponent is close to the best value [5] accepted for the two-dimensional Ising model using the Metropolis prescription. We believe that this value is the first estimate of the dynamical critical exponent for a quantum spin model.

V. CONCLUSIONS

We have used the short-time scaling law to determine some critical exponents of the transverse Ising model in one dimension at zero temperature. The Monte Carlo method employed to obtain the relaxation of the order parameter already at the initial stages of the evolution towards the thermodynamical equilibrium was recently introduced and is appropriated to obtain the ground-state properties of quantum spin models. To the best of our knowledge, an estimate of the dynamical critical exponent z for quantum models has not been obtained until now. On the other hand, the values found for the static critical exponents are in agreement with the exact ones.

ACKNOWLEDGMENTS

I wish to acknowledge Dr. Wagner Figueiredo and Dr. Luis G. C. Rego for their reading of the manuscript and their critical comments. This work was supported by the Brazilian agency CNPq.

[1] H.K. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B: Condens. Matter **73**, 539 (1989).
[2] Z.B. Li, L. Schulke, and B. Zheng, Phys. Rev. Lett. **74**, 3396 (1995).
[3] Z.B. Li, L. Schulke, and B. Zheng, Phys. Rev. E **53**, 2940 (1996).
[4] L. Schulke and B. Zheng, Phys. Lett. A **233**, 93 (1997).
[5] Z.B. Li, X.W. Liu, L. Schulke, and B. Zheng, Physica A **245**, 485 (1997).
[6] A. Jaster, J. Mainville, L. Schulke, and B. Zheng, J. Phys. A **3**, 1395 (1999).
[7] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).
[8] P. Pfeuty, Ann. Phys. (N.Y.) **57**, 79 (1970).
[9] M.J. de Oliveira, Phys. Rev. B **48**, 6141 (1993).
[10] R.A. Sauerwein and M.J. de Oliveira, Phys. Rev. B **49**, 5983 (1994).
[11] R.A. Sauerwein and M.J. de Oliveira, Mod. Phys. Lett. B **9**, 619 (1995).
[12] J.P. Neirotti and M.J. de Oliveira, Phys. Rev. B **53**, 668 (1996).
[13] M.J. de Oliveira and J.R.N. Chiappin, Physica A **238**, 307 (1997).
[14] M. Santos and M.J. de Oliveira, Int. J. Mod. Phys. B **13**, 207 (1999).
[15] M. Suzuki, Prog. Theor. Phys. **58**, 1142 (1977).
[16] R.J. Glauber, J. Math. Phys. **4**, 294 (1963).